The Complete Book of

Trigonometric Identities

And

Unit Circle Values

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Author's Note

Trigonometry has been a passion of mine since the late months of 2021. When the unit circle was introduced in my honor's algebra II class, my life was forever changed. Since then, I have taken a similar liking to trig identities; they feel like fun puzzles like sudoku and word search! I truly hope you get as much enjoyment as I do from the trigonometry in this book.

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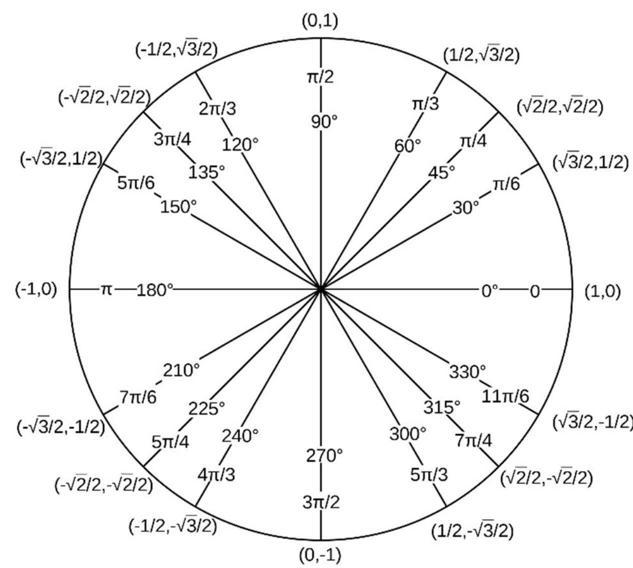
Finding Values on the Unit Circle



$\sin\left(\frac{5\pi}{6}\right)$	$\sin\left(\frac{3\pi}{4}\right)$	$\sin\left(\frac{23\pi}{6}\right)$	$\sin\left(\frac{12\pi}{4}\right)$
$\cos\left(\frac{3\pi}{2}\right)$	$\cos\left(\frac{7\pi}{4}\right)$	$\cos\left(\frac{7\pi}{2}\right)$	$\cos\left(\frac{16\pi}{4}\right)$
$\tan\left(\frac{2\pi}{3}\right)$	$\tan\left(\frac{\pi}{6}\right)$	$\tan\left(\frac{20\pi}{3}\right)$	$\cot\left(\frac{37\pi}{6}\right)$
$\sec\left(\frac{11\pi}{6}\right)$	$\sec\left(\frac{5\pi}{4}\right)$	$\sec\left(\frac{13\pi}{6}\right)$	$\sec\left(\frac{21\pi}{4}\right)$
$\csc\left(\frac{7\pi}{6}\right)$	$\csc\left(\frac{\pi}{3}\right)$	$\csc\left(\frac{31\pi}{4}\right)$	$\csc\left(\frac{19\pi}{4}\right)$

$\sin\left(\frac{5\pi}{12}\right)$	$\sin\left(\frac{7\pi}{5}\right)$	$\sin\left(\frac{8\pi}{7}\right)$	$\sin\left(\frac{9\pi}{10}\right)$
$\cos\left(\frac{6\pi}{10}\right)$	$\cos\left(\frac{9\pi}{14}\right)$	$\cos\left(\frac{3\pi}{20}\right)$	$\cos\left(\frac{7\pi}{8}\right)$
$\tan\left(\frac{2\pi}{7}\right)$	$\cot\left(\frac{\pi}{15}\right)$	$\tan\left(\frac{2\pi}{13}\right)$	$\cot\left(\frac{\pi}{10}\right)$
$\sec\left(\frac{11\pi}{5}\right)$	$\sec\left(\frac{13\pi}{12}\right)$	$\sec\left(\frac{2\pi}{16}\right)$	$\sec\left(\frac{9\pi}{5}\right)$
$\csc\left(\frac{3\pi}{8}\right)$	$\csc\left(\frac{3\pi}{13}\right)$	$\sin\left(\frac{6\pi}{11}\right)$	$\sin\left(\frac{3\pi}{7}\right)$

Trigonometric Identities

$$sin(\theta)$$

~~~

$$cos(\theta)$$
~~~

Reciprocal Identities:

$\sec(\theta) = \left(\frac{1}{\cos(\theta)}\right)$ $\csc(\theta) = \left(\frac{1}{\sin(\theta)}\right)$ $\cot(\theta) = \left(\frac{1}{\tan(\theta)}\right)$

$\tan(\theta)\sin(\theta) + \cos(\theta)$

$\cot(\theta)\tan(\theta)$

$\sin(\theta)\sec(\theta)$

$\cos(\theta)\csc(\theta)$

$\tan(\theta)\csc(\theta)$

$\tan(\theta)\cos(\theta)\csc(\theta)$

$\sin(\theta) + \cot(\theta)\cos(\theta)$

$$\frac{\sec(\theta)}{\csc(\theta)}$$

$$\frac{\sin(\theta)}{\csc(\theta)} + \frac{\cos(\theta)}{\sec(\theta)}$$

$$\frac{\cot(\theta)}{\csc(\theta)-\sin(\theta)}$$

$$\frac{1+\cot(\theta)}{\csc(\theta)}$$

$$\frac{1+\csc(\theta)}{\cos(\theta)+\cot(\theta)}$$

$$\frac{\sin(\theta)\sec(\theta)}{\tan(\theta)}$$

$$\frac{\sec(\theta)-\cos(\theta)}{\tan(\theta)}$$

$$\frac{\cos(\theta)\sec(\theta)}{\tan(\theta)}$$

$$\frac{\tan(\theta)}{\sec(\theta)}$$

$$\frac{\sin(\theta)}{\tan(\theta)}$$

$$\frac{\cot(\theta)\sec(\theta)}{\csc(\theta)}$$

Pythagorean Identities:

$$sin^2(\theta) + cos^2(\theta) = 1$$
$$1 + tan^2(\theta) = \sec(\theta)$$
$$1 + cot^2(\theta) = \csc(\theta)$$

$tan^2(\theta) - sec^2(\theta)$

$cos^2(\theta)(1 + tan^2(\theta))$

$cos^3(\theta) + sin^2(\theta)\cos(\theta)$

$\frac{1}{1-sin^2(\theta)}$

$sin^4(\theta)cos^4(\theta)cos^2(\theta)$

$\frac{sec^2-1}{sec^2}$

$\frac{2+tan^2}{sec^2} - 1$

$(1 - \cos(\theta))(1 + \cos(\theta))$

$tan^2(\theta) - cot^2(\theta) = sec^2(\theta) - csc^2(\theta)$

$(tan(\theta) + \cot(\theta))^2 = sec^2(\theta) + csc^2(\theta)$

$tan^2(\theta) - sin^2(\theta) = tan^2(\theta)sin^2(\theta)$

$sec^4(\theta) - tan^4(\theta) = sec^2(\theta) + tan^2(\theta)$

$\sec(\theta)\csc(\theta)(\tan(\theta) + \cot(\theta)) = sec^2(\theta) + csc^2(\theta)$

Even/Odd Identities

$\sin(-\theta) = -\sin(\theta)$ $\quad \csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$ $\quad \sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\quad \cot(-\theta) = -\cot(\theta)$

$$\frac{\tan(\theta)}{\sec(-\theta)}$$

$$\tan(\theta) + \cos(-\theta)$$

$$(\cot(\theta) - \csc(\theta))(\cos(\theta) + 1)$$

$$\frac{\csc(\theta)}{1+cot^2(\theta)}$$

$$\sin(\theta)(\tan(\theta) + \cot(\theta))$$

$$9 - cos^2(\theta) - sin^2(\theta)$$

Sum & Difference Identities

$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$$\sin(x + y) - \sin(x - y) = 2\cos(x)\sin(y)$$

$$\cos(x + y) - \cos(x - y) = 2\cos(x)\cos(y)$$

$$\cos(x + y)\cos(x - y) = cos^2(x) - sin^2(y)$$

$$\sin(x-\pi) = -\sin(x)$$

$$\tan(x) - \tan(y) = \frac{\sin(x-y)}{\cos(x)\cos(y)}$$

$$\cos(3\theta) + \cos(\theta) = 2\cos(2\theta)\cos(\theta)$$

$$\sin(4\theta) + \sin(2\theta) = 2\sin(3\theta)\cos(\theta)$$

$$\sin(3\theta) = 3cos^2(\theta)\sin(\theta) - sin^3(\theta)$$

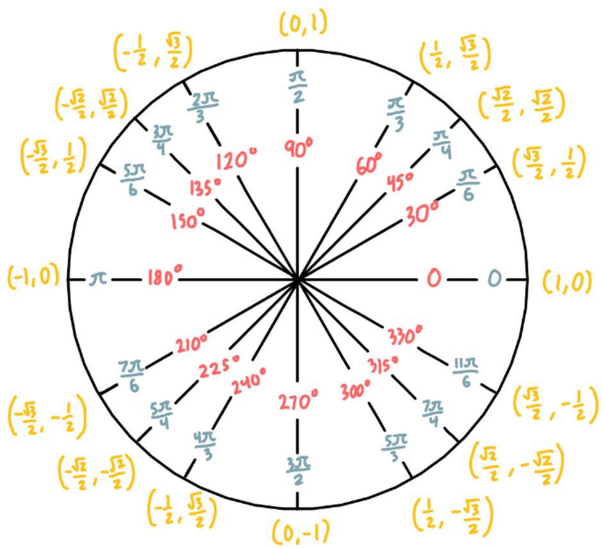
$$\sin(2\theta)\cos(\theta) = \cos(2\theta)\sin(\theta)$$

$$\cos(3\theta)\cos(\theta) = \sin(3\theta)\sin(\theta)$$

$$\cos(3\theta) = cos^3(\theta) - 3sin^2(\theta)\cos(\theta)$$

Complete Unit Circle Diagram

(With explanations/tricks)



Degrees: the key for looking at degrees is really by mainly memorizing the values on each axis, and building from there based on which quadrant your desired value is in.

Ex: looking for **135**°

1. Determine which quadrant your angle is in by thinking

2. what two axis points it is
3. between (**90°** and **180°** -> second quad.)
4. If the value has a "5" in it, use the middle line to place it
5. If the value is one of the other two, place it on the line closer to **90°** or **180°** based on what it's closer to

Radians: the key for looking at radians mainly relies on memorizing the placement of the first $\pi/6$, and $\pi/4$ and add/reduce fractions from there.

Ex: looking for $2\pi/3$

1. Locate $\pi/6$ and start counting around on all lines but the "middle of quadrant" ones, adding as you go
2. Reduce $4\pi/6$ to $2\pi/3$

Ex: looking for $\pi/4$

1. Repeat the same process, only using the "middle of quadrant" lines and axis lines

Points: the key for looking at points truly only requires memorizing two points: any one of the points on the "non-middle/non axis lines," and $\boldsymbol{\pi/4}$. From there, simply remember these rules:

For the first group of points, if the point you are looking for is in the same quadrant as your memorized point, then the values will swap positions: $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \rightarrow \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. If the value is across an axis, the positions stay the same, but +/- signs switch depending on the quadrant

For the second group of points, know that any point on a variation of $\boldsymbol{\pi/4}$ will always have a value of $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ with +/- signs varying based on quadrant

For the third group, consider these axis points in terms of the radius. If it's an x-axis, the value will be (1,0), with the radius in the x position, and if on a y-axis, it will be in the y position, (0,1). Change +/- signs based on quadrant

Glossary of All Trigonometric Identities

(With explanations/tricks)

Reciprocal Identities:

$\sec(\theta) = \left(\frac{1}{\cos(\theta)}\right)$

$\csc(\theta) = \left(\frac{1}{\sin(\theta)}\right)$

$\cot(\theta) = \left(\frac{1}{\tan(\theta)}\right)$

you can also view **cot as just **cos/sin***

Pythagorean Identities:

$sin^2(\theta) + cos^2(\theta) = 1$

$1 + tan^2(\theta) = \sec(\theta)$

$1 + cot^2(\theta) = \csc(\theta)$

derive the singular **sec and **csc**, as well as the **sin** and **cos** versions of these identities by solving for the desired value from the above identities*

Even/Odd Identities:

$\sin(-\theta) = -\sin(\theta)$

$\csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$

$\cot(-\theta) = -\cot(\theta)$

**Essentially, for memorization, just recall that the reciprocal identities for each rule will follow the same format as the "original."*

Sum and Difference Identities:

$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

$$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$

(Sum/Difference Cont.) For **sin identities, the +/- sign stays consistent with the left side of the equation, and pairs of **sin** and **cos** mix.*

*For **cos** identities, the +/- sign is the opposite of the left side of the equation, and pairs of **sin** and **cos** go together*

*For **tan** identities, the additive piece goes on the top, and the multiplicative on the bottom with a "1." The +/- sign on the left side of the equation stays consistent on the top, and switches on the bottom.*

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